‘That There Might Be Vague Objects (So Far as Concerns Logic)*

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Realism about vagueness is anti-realism about the world.
—Michael Dummett

1 Opening

Some years ago, Gareth Evans argued, in a well-known paper (Evans, 1978), that there can be no vague objects. Evans's paper has been the subject of much discussion, but little agreement has been reached about his argument: There is little agreement regarding what is in dispute, what sorts of arguments might decide it, how Evans's argument addresses the problem, or what objections to that argument are relevant. I shall attempt here to resolve some of these difficulties. First, I shall look at what principles are required if Evans's formal argument is to succeed; I shall then consider objections to some of them, all of which I shall dismiss. I shall thus be arguing that Evans's formal argument is valid, but shall yet reject the ultimate conclusion of Evans's paper, that the argument shows that there can be no vague objects. For Evans presupposes that an object is vague just in case some identity-statements concerning the object have a quite specific semantic property. I shall argue that the vagueness of an object need not be so understood, that there is a weaker semantic property which, logically speaking, identity-statements concerning an object might possess, and that possession of this weaker property plausibly constitutes the vagueness of the object in question. I shall not, however, attempt to decide the question whether there are vague objects: The conclusion of the present paper is just that logic alone does not preclude the existence of such objects.

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My purpose is not entirely negative, however. Our discussion will reveal that there is a view naturally described as committed to the existence of vague objects which has, to this point, not even been considered. Moreover, examination of Evans's argument will reveal how important it is, when discussing vagueness, to respect the distinction between axioms and rules of inference;[1] moreover, properly to formulate the view that there are vague objects, one must distinguish the unsatisfiability of a formula from the validity of its negation. Neither of these distinctions is substantial in the case of classical logic, and they are therefore frequently ignored. But it should hardly come as a surprise that, if one's goal is a logic of vagueness, the logic will almost inevitably be non-classical, whence one must be prepared for all sorts of wild and wonderful phenomena.[2] The *sort* of logic in which Evans's argument is carried out strikes me as an entirely natural one for vagueness, formally speaking. The semantics of such logics are another matter, about which I am able to say little, but they are amenable to relatively familiar sorts of model-theoretic treatment.

2 Evans's Formal Argument

Before beginning the discussion of Evans's argument, it is worth reminding ourselves of its formal part. Where '∇', which one might call atled, is an operator read "It is indeterminate whether...", the argument is, in short:[3]

1. $\nabla(a = b)$
2. $\neg\nabla(a = a)$
3. $b = a \wedge \nabla(a = b) \rightarrow \nabla(a = a)$

[1] I have also argued for this elsewhere (Heck, 1993). The argument there does not concern vague objects but a puzzle, due to Crispin Wright, concerning higher-order vagueness.

[2] The non-classical character of the logic need not reside in the invalidity of any classically valid schema, but may instead result from the presence of non-classical operators. Indeed, in every section of the present paper, except Section 5, we will simply be presupposing the validity of classical predicate logic.

[3] Evans's original presentation of the argument involves the use of predicate-abstraction. After much reflection on remarks made by Richard Cartwright, I have become convinced that this both invites confusion and is inessential. I first heard the term 'atled' from George Boolos. I take it that it is a neologism.

4. $b \neq a$

Or, informally: Suppose that it is indeterminate whether b is a. It is not indeterminate whether a is a, for it is perfectly determinate whether (indeed that) a is a. But if b were a, then, since it is indeterminate whether a is b, it would have to be indeterminate whether a was a. Therefore, b is not a.

Evans remarks, after the completion of the formal argument, that this conclusion contradicts the assumption with which we began, that it is indeterminate whether a is b. I shall return below to the question why this is so. I shall also return to the question exactly what the argument is meant to establish.

Evans's argument plainly relies upon a number of different principles. First, it relies upon the principle of the Indiscernibility of Identicals. For the moment, we may assume that Evans would maintain the validity of the schema:

(LL) $x = y \wedge \Phi(x) \rightarrow \Phi(y)$

This schema is, of course, equivalent to the following one:

(II) $\Phi(x) \wedge \neg\Phi(y) \rightarrow x \neq y$

Such a principle justifies the transition from '$\neg\nabla(a = a)$' and '$\nabla(a = b)$' to '$b \neq a$'.

Second, the application of this principle rests upon the claim that the operator '∇' does not induce an opaque context, so that '$\nabla(a = \xi)$' is a permissible substituend for '$\Phi(\xi)$' in (LL).

Third, Evans relies upon the claim that reflexive identities are not of indeterminate truth-value. Where 'Δ' is an operator to be read "It is determinate whether. . . ", we may record the principle as:

(R) $\Delta(a = a)$

Fourthly, if it is determinate whether A, it is not indeterminate whether A:

(C′) $\Delta A \rightarrow \neg\nabla A$

From (R) and (C), we may infer Evans's third premise.

These are the only assumptions appeal to which is required for the formal argument in Evans's paper, but it is worth nothing that Evans would in fact seem to accept the stronger claim that it is determinate whether A if, and only if, it is not indeterminate whether A. We record this as:

(C) $\Delta A \equiv \neg\nabla A$

Evans also remarks that '∇' and 'Δ' are "duals". We should thus probably ascribe the following principle to him:

(D) $\Delta A \equiv \neg\nabla\neg A$

In any event, the following is surely valid:

(Eq) $\Delta A \equiv \Delta(\neg A)$

For, if it is determinate whether A, surely it is also determinate whether not-A. It does not in fact matter which two of (C), (D), and (Eq) one takes as axioms, since each of the three is derivable from the other two.

Now, a great deal of confusion has been caused by a slip in Evans's paper.[4] Evans there equivocates, confusing the operator 'Δ' with a related but distinct operator '$\Box$', to be read "It is definite(ly the case) that…". The principle

(T$\Box$) $\Box A \to A$

is a natural one; we may take

(D$\Box$) $\Diamond A \equiv \neg\Box\neg A$

as the definition of a dual operator. Principles analogous to (C) and (Eq), however, are plainly invalid: If it is definite that A, not only does it not follow that it is definite that not-A, it follows that it is *not* definite that not-A.

If we do not keep these operators separate, we are going to have some problems. At one point in his paper, Evans appeals to the principle:

(T) $\Delta A \to A$

As was said, the analogue of this principle is valid for the operator "Definitely". But, given the interpretation of 'Δ' as "It is determinate whether…", (T) is plainly invalid: If it is determinate whether A, it does not follow that A is true; A may be either determinately true or determinately false. And it is easy to derive, from (Eq) and (T), that

[4]Pelletier *(1989, p. 482, fn)* says that, according to Lewis, Evans retracted this in correspondence. (Since most of my discussion of it will be critical, I should like to note my debt to Pelletier's paper, for it was it which sparked my interest in Evans's argument.)

'$\neg\Delta$A' is valid.[5] In the presence of (R), or of any principle asserting that some sentence is of determinate truth-value, contradiction is immediate. Note, however, that the operators '$\Box$' and 'Δ 'are closely related. Given an operator '$\Box$' , like "Definitely", for which (T$\Box$) and (D$\Box$) are valid, we can define an operator 'Δ' for which (Eq), (C), and (D) are valid:

$$\Delta A \stackrel{df}{\equiv} \Box A \wedge \Box\neg A$$

$$\nabla A \stackrel{df}{\equiv} \neg\Delta A \equiv \Diamond A \wedge \Diamond\neg A$$

(Eq) is then obvious;[6] (C) is just the definition of '∇'; and, (D) follows from (Eq) and (C). Conversely, given an operator 'Δ', we can define an operator '$\Box$' satisfying (T$\Box$) and (D$\Box$):

$$\Box A \stackrel{df}{\equiv} \Delta A \wedge A$$

$$\Diamond A \stackrel{df}{\equiv} \neg\Box\neg A \equiv \neg A \vee \nabla A$$

(T$\Box$) is then obvious, and (D$\Box$) is the definition of the dual. But analogues of (C) and (Eq) are plainly invalid.

Operators akin to "It is determinate whether..." and "It is definite that..." are thus interdefinable. The reading of 'Δ' as "It is determinate whether..." may now be further explained: To say that it is determinate whether A is to say that either it is definite that A or it is definite that not-A. Since operators such as "Definitely" are rather more often discussed in connection with vagueness, perhaps this reading is more helpful than the official interpretation with which I began.

3 What Evans Argued

For the purposes of our discussion here, I shall assume, as earlier, that Evans would hold the principles (C), (D), and (Eq), as well as (R), to be

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[5] As mentioned in note 4, Pelletier (1989, pp. 483-4) notes Evans's retraction of this slip, but he misses this point: His attempt to derive a contradiction from '$\nabla(a = b) \rightarrow a \neq b$' is invalidated precisely by an appeal to (T). In fairness to Pelletier, he does note that some of the writers he is criticizing hold that (T) is valid; but I presume that what they accept is (T$\Box$).

For purposes of typographical simplicity, I shall, as here, be sloppy about quasi-quotation.

[6] Given that classical logic is in play—in particular, that '$\neg\neg A$' is equivalent to 'A'—and that '$\Box$' respects that equivalence, as is certainly the case here.

valid. For ease of exposition, let me refer to one who maintains, with Evans, that there can be no vague objects as a *Definitist*; h'er opponent, as an *Indefinitist*.

Evans assumes, *for the purposes of argument*, that the operator '∇' does not induce an opaque context. As Evans conceives it, the ontological or metaphysical view that there are vague objects must have a semantic component; indeed, Evans probably would have identified the view that there are vague objects with a certain view about the functioning of names which apparently denote vague objects. What Evans is arguing is not that no identity-statement is vague, but rather that, whenever a sentence of the form '$a = b$' is of indeterminate truth-value, it is so only because one or both of the *terms* 'a' and 'b' is vague, because it is indeterminate to what one of these terms refers. Indefinitism is thus not characterized by the claim that some identity-statements are vague. Rather, what distinguishes it is the claim that some identity-statements are of indeterminate truth-value *not* because it is indeterminate to what the relevant terms refer, but because the objects to which they refer are *themselves* indeterminate (or vague). To hold this view, to hold that there are vague objects, is to hold that the vagueness of a statement about such an object may be a consequence, not of how one refers to it, but rather of the nature of the object itself. That is, whether statements of the form 'Fa' are of determinate truth-value *may* (not 'must') depend, in respect of the term 'a', *only* upon to what 'a' refers. On this view, the relevant application of Leibniz's Law cannot be denied, if it is supposed that 'a' and 'b' are terms which denote objects which are themselves indeterminate, rather than terms which indeterminately denote objects which are themselves not vague.

Thus, an Indefinitist cannot deny the relevant application of Leibniz's Law on the ground that '$\Box$' and 'Δ' are opaque, since she must hold that the determinacy (or definiteness) of a given identity-statement may depend only upon the nature of the objects—as determinate or vague—referred to. She must, instead, concede that these operators are *transparent*. Note, however, that the transparency of '∇' does not guarantee the validity of the relevant application of (LL); this depends, also, upon the assumption that (LL) is valid in general. An operator 'Ω' is (as I use the term) transparent if, whenever '$A(\xi)$' is a legitimate substituend of '$\Phi(\xi)$' in (LL) *or whatever analogous formal principle might be supposed to govern identity*, so is '$\Omega A(\xi)$'. Hence, if (LL) is valid, then '$\Box$' (or 'Δ')

will not only be transparent but will be *extensional*, in the sense that

$$x = y \rightarrow Fx \equiv Fy$$

is valid. But there might be reason to question the validity of (LL), without questioning whether (say) the predicate 'ξ is definitely red' is satisfied by *objects*, that is, without questioning the transparency of '$\Box$'. I shall return to this in Section 6.

The argument here given on behalf of Evans's assumption of the validity of his application of (LL) can be illuminated by comparing it with remarks made by David Lewis (1988). Lewis writes that only one who holds that a vague name does not "rigidly [denote] a vague object" can balk at Evans's application of (LL). To see the point of this remark, consider the argument, due to Quine, which purports to show that every extensional operator is truth-functional; in application to this case, its conclusion would be that '$\Box$' is truth-functional. The argument relies upon the supposition that mathematical equivalents can be substituted for one another inside '$\Box$' and may, in the present context, be formulated as follows:[7]

1.	$p \equiv q$	Premise
2.	$\Box p$	Premise
3.	$\Box(0 = \{x : x = 0 \wedge p\})$	Substitution of provable equivalents
4.	$\Box(0 = \{x : x = 0 \wedge q\})$	Leibniz's law, extensionality, etc.
5.	$\Box q$	Substitution of provable equivalents
6.	$p \equiv q \wedge \Box p \rightarrow \Box q$	Conditional proof

In order to resist this argument, we need, just as in quantified modal logic, to distinguish terms which designate vague objects 'rigidly' from those which 'non-rigidly' designate possibly quite determinate objects. Only 'rigid' designators of the same object will be allowed to be substituted for one another inside the scope of '$\Box$'. Thus, just as in the case of quantified modal logic, the step of the argument which is disallowed is the application of Leibniz's Law.

[7]In the logics of vagueness to be developed below, the *inference* from '$p \equiv q$' and '$\Box p$' to '$\Box q$' is going to be *valid*, but the conditional '$p \equiv q \wedge p \rightarrow q$' will not be. It is thus the invalidity of this conditional for which we need to account.

To put the point less formally, if it is indeterminate whether p, it will certainly be indeterminate whether "$\{0 : x = 0 \wedge p\}$" denotes the null set or instead denotes $\{0\}$. But there is no need to hold that there is some set which is *itself* indeterminately either the null set or $\{0\}$. This term is not one which designates a vague object, but one which vaguely designates: Nor is there any reason that the terms "$\{0 : x = 0 \wedge p\}$" and "$\{0 : x = 0 \wedge q\}$" must designate the same object if made more precise. (If we knew that the vagueness of 'p' was precisely matched by that of 'q'—i.e., if we knew that $\Box(p \equiv q)$—there would be no problem.) Hence, to deny that any terms 'rigidly' designate vague objects is to deny that there are any terms which denote objects which are *in themselves* vague, while accepting that it may be vague what a term denotes.

To summarize: Indefinitism cannot properly be characterized as the view that some identity-statements are vague. Lots of people (including Evans) believe that some identity-statements are vague: But many of these people (including Evans) believe that the vagueness of such statements is a product, not of the vagueness of the objects themselves, but of our *language* (or, more generally, our thought). An Indefinitist must also hold that operators such as "It is determinate whether…" may be transparent and that, in such a sense, not every identity-statement is of determinate truth-value, even if there is no indeterminacy concerning to what the relevant expressions refer: Only if such an operator may be transparent can it be said that the truth-value of a sentence containing it (and so the vagueness of that sentence) depends, not upon how the objects to which we refer are 'described', but rather upon the nature of the objects themselves.

We may conclude that it is not to respond to Evans, but to concede his point, to claim that "It is indeterminate whether…" cannot be, and any similar operator would not be, transparent. If so, the indeterminacy of a given statement depends upon how we refer to the objects to which we refer; and that is to say that the vagueness of the statement is a product not of reality but of language.

Given this account of what an argument for Definitism must accomplish, we may formulate a simple restriction upon such arguments. To defeat Indefinitism, what one must show is that, if 'Δ' is an operator which can plausibly be construed as "It is determinate whether…", then, if 'Δ' is transparent, no identity-statement is of indeterminate truth-value. But the argument must therefore show that there is a *special* problem which arises if we treat 'Δ' as an transparent operator, and it must show that there is some special problem about *identity*.

To contrapose: If 'ΔA' is valid, then we have been given no argument for Definitism, since 'Δ' cannot plausibly be read as "It is determinate whether…".

4 Whence the Contradiction?

The question now before us is whether Evans's argument, the assumptions made thus far being granted, establishes his claim. I am granting that the formal argument Evans sets out is one which must be accepted by an Indefinitist. The question is whether the Indefinitist is committed to the truth of a contradiction.

Evans argues, recall, that we can derive the conclusion '$a \neq b$' from the assumption '$\nabla(a = b)$', which, he says, it contradicts. It is not obvious why this should be so. We may take Evans to have meant that, if '$a \neq b$' is true, then '$a \neq b$' is of determinate truth-value. Hence, there would seem to be an unrecorded step in Evans's argument, from '$a \neq b$' to '$\Delta(a = b)$', which, in the presence of (D), does contradict '$\nabla(a = b)$'. But to what principle is Evans appealing here? What justifies this inference?

The simplest principle to which we might take Evans to be appealing is:

(Det) $A \rightarrow \Delta A$

However, if he means to appeal to this principle, then his argument might have avoided questions of identity altogether. Viz.:

1. $A \rightarrow \Delta A$ (Det)
2. $\neg A \rightarrow \Delta\neg A$ (Det)
3. $\Delta A \equiv \Delta\neg A$ (Eq)
4. $\neg A \rightarrow \Delta A$ (2,3) TF
5. ΔA (1,4) TF

If Evans intends to appeal to (Det), he has no argument against the existence of vague objects. For, as argued in the last section, since we can show that 'ΔA' is valid, the theory in question—(Det)+(C)+(Eq)—is inadequate as a theory of vagueness (or lack thereof), as no statement, in the sense of this operator, can possibly be vague.

It might also be suggested that Evans intends to appeal to some claim regarding the modal logic of 'Δ'. He writes that, "if Δ determines a logic at least as strong as S5", then '$\Delta(a \neq b)$' is derivable from '$\nabla(a = b)$'.[8] As a version of the characteristic axiom of S5, i.e., '$\Diamond A \to \Box\Diamond A$', we may take:[9]

(5Δ) $\nabla A \to \Delta\nabla A$

We also need to appeal to the following distribution axiom:[10]

(Dist) $\Delta(A \to B) \to (A \wedge \Delta A \to \Delta B)$

Finally, we need the following analogue of necessitation

(NecΔ) If A is valid, so is 'ΔA'.

The argument is then as follows:

1. $\nabla(a = b) \to \neg(a = b)$ — Evans's argument, conditional proof
2. $\Delta(\nabla(a = b) \to \neg a = b)$ — (1) Nec Δ

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[8]As Bob Stalnaker suggested to me, this remark is partly the result of the 'slip' mentioned earlier, so Evans may most charitably be interpreted as discussing not 'Δ' but '$\Box$'.

[9]This principle can be derived, given 'FIXME $A \to A$', from the more usual S5 axiom mentioned.

[10]The analogue of the standard distribution principle, which allows the inference from '$\Delta(A \to B)$' to '$\Delta A \to \Delta B$', is invalid, if any sentence whatsoever is of indeterminate truth-value. To see this, let A be the falsum. Then '$\bot \to B$' is valid, so '$\Delta(\bot \to B)$' is valid; similarly, '$\bot$' is always false, so '$\Delta\bot$ ' is also valid; but then 'ΔB' must be valid, whatever B is. (Thanks to David Lewis for this point.)

To prove the restricted distribution principle in the text, use the equivalences mentioned above between '$\Box A$' and '$A \wedge \Delta A$', and between 'ΔA' and '$\Box A \vee \Box\neg A$'. Thus:

1. $\Delta(A \to B)$ — Premise
2. $\Box(A \to B) \vee \Box\neg(A \to B)$ — $\Delta/\Box$ equivalences
3. $(\Box A \to \Box B) \vee \Box(A \wedge \neg B)$ — $\Box$-Distribution, TF
4. $(\Box A \to \Box B) \vee \Box A \wedge \Box\neg B$ — $\Box$-Distribution
5. $\Box A \to (\Box B \vee \Box\neg B)$ — (4)TF
6. $\Box A \to \Delta B$ — $\Delta/\Box$ equivalences
7. $A \wedge \Delta A \to \Delta B$ — $\Delta/\Box$ equivalences
8. $\Delta(A \to B) \to (A \wedge \Delta A \to \Delta B)$ — Conditional proof

Note that the rule to be called (V) below is not used in this argument, so the step at line (8) is still legitimate in the context of logics containing it.

3. $\nabla(a = b) \wedge \Delta\nabla(a = b) \to \Delta(\neg a = b)$ (2), Dist

4. $\nabla(a = b) \to \Delta\nabla(a = b)$ (5Δ)

5. $\nabla(a = b) \to \Delta(\neg a = b)$ (3,4) TF

6. $\nabla(a = b) \to \Delta(a = b)$ (5), Eq

7. $\neg\Delta(a = b) \to \nabla(a = b)$ C

8. $\Delta a = b$ (6,7) TF

That would indeed establish Definitism.

But what sort of justification can be given for (5Δ)?[11] The most natural is the following. Suppose that there are exactly three truth-values, True, False, and Neither. We may take 'ΔA' to be True if A is either True or False; otherwise, it is False. Similarly, '∇A' is True if A is Neither, otherwise False. Then we can justify (5Δ): By definition, '∇A' must be either True or False; either way, '$\Delta\nabla A$' is True, whence '$\Delta\nabla A$' is valid, and so (5Δ) is valid.

This justification of (5Δ), however, is one which the Indefinitist has no reason to accept. Recall that we may define an operator '$\Box$' as follows:

$$\Box A \overset{df}{\equiv} A \wedge \Delta A$$

Note, then, that '$\Box A$' is always either True or False: For A is either True, False, or Neither. If A is True or False, then 'ΔA' is True; so '$\Box A$' is True or False, as A is True or False. Similarly, if A is Neither, then 'ΔA' is False, so '$\Box A$' is False. Hence: '$\Box A$' is True if, and only if, A is True; otherwise, it is False. The offered justification for (5Δ) thus also provides a justification for this principle, which is in fact formally derivable from (5Δ), (T), and the definition of '$\Box$':[12]

$$\Delta\Box A$$

[11] I should thank Bob Stalnaker for suggesting this as a possible justification; his suggestion greatly improved this section of the paper. In previous drafts, I was rather at a loss for a justification, since I was concentrating upon the equivalent axiom '$\nabla\Delta A \to \Delta A$', for which it is very difficult to give any intuitively compelling reason. Nonetheless, it is valid, on the suggested interpretation of 'Δ', since the antecedent is, as we shall see, unsatisfiable.

[12] The formal derivation is space-consuming, since 'Δ' is the primitive operator. If we introduce '$\Box$' as primitive and define 'Δ' as earlier, the proof is rather easier. For '$\Delta\Box A$' is equivalent to 'FIXME", i.e., to 'FIXME', which is provable in S5. (Note that the discussion of (5Δ) above actually showed it to be valid, too.)

So, on this treatment, there can be no higher-order vagueness: Even if A is vague, 'Definitely: A' cannot be. That is to say, using '$\Box$', we can speak about our (by hypothesis) vague subject matter *with no vagueness whatsoever*. All we need do is take care to insert '$\Box$' before anything we write or say, and whatever vagueness might have affected our utterance will be removed: None of our utterances will fail to be determinately true or false.

At the very least, an Indefinitist has no obligation to accept a principle that logically precludes the existence of higher-order vagueness.[13] I think that something stronger may be true—namely, that an Indefinitist not only has reason to reject any logical principle that *excludes* higher-order vagueness, but that s'he is, in general, *committed* to the existence of higher-order vagueness. Surely the picture proposed, that there is vagueness in reality— *any* sort of vagueness, whether that of properties or of objects—could hardly be explained better than as follows: The vagueness which characterizes our talk about such objects is an essential feature of it, one which cannot be eliminated by the introduction of as-yet-unheard-of operators into the language; it is not our language which is responsible. Vagueness, as one might put it, is *ineradicable* if there is vagueness 'in reality'. But the semantic assumptions used to justify (5Δ) are strong enough to justify the introduction of operators which eradicate vagueness; and, quite independently of those assumptions, the validity of '$\Delta\Box A$', which is implied by that of (5Δ), surely implies the eradicability of vagueness. Since both the semantic assumptions and the formal principle (5Δ) imply that vagueness is eradicable, the assumption of either begs the question against the Indefinitist.

The problem with these remarks is that the claim that vagueness is ineradicable, as it stands, is rather imprecise. The claim cannot be that *no* operator can elimiate vagueness, as the trivial falsum operator would surely do that. The thought, rather, is that, while it is almost essential to such views that there are operators which *strengthen* vague state-

[13] Pelletier's argument, in terms of many-valued logic, relies upon essentially the same claim: His J-operators may be defined in terms of "$\Box$', subject to S5. In a slightly different, but self-explanatory, terminology: FIXME... It is then not too difficult to derive a contradiction from FIXME. See Pelletier, pp. 488-90.

Similar problems will affect many-valued treatments in terms of any finite number of truth-values. In each such case, the logic will preclude the existence of an operator which is n^{th}-order vague, for some n; and an Indefinitist has no reason to accept that logic limits the number of orders of vagueness.

ments, which make them rather less vague (for example, "Definitely"), there can be no such operator which *eliminates* vagueness: If A is vague, so in general is "Definitely:[14] A".[15] But what is 'such' an operator? what it is for an operator to 'strengthen' a vague statement in the relevant sense? At present, I have no very substantial proposal to make about this.[16]

The issue can, however, be set aside, for we have already seen that the Indefinitist can reject (5Δ) and the justification offered for it simply on the ground that they *preclude* higher-order vagueness and does not need the stronger claim that vagueness is ineradicable, however that might be developed. And, in any event, it is hardly likely that Evans intended to appeal to this sort of modal principle. For he says that "*if* Δ determines a logic at least as strong as S5", then '$\Delta a = b$' is derivable from '$\nabla(a = b)$'". Appeal to such a principle is no part of his argument. The fact that we can derive '$a \neq b$' from '$\nabla(a = b)$' is the problem: The remark about S5 is but an aside. I pursue it only to show that this avenue is definitely closed.

But we ought to be extremely puzzled by this last mentioned remark from Evans's paper. Surely, if, as Evans says, '$a \neq b$' *contradicts* the assumption that '$\nabla(a = b)$', then '$a \neq b$' must be at least as strong as '$\neg\nabla(a = b)$': If one statement *contradicts* another, then it must imply the negation of that other statement. If so, then, for whatever reason, '$\neg\nabla(a = b)$', i.e., '$\Delta a = b$', must follow from '$a \neq b$'. But why then does Evans say that it is only *if* the logic governing 'Δ' is at least as strong as S5 that we can derive '$\Delta a = b$' from '$a = b$'? To this question, I can give no sure answer: But I suspect that Evans was struggling to express a quite different, and ultimately crucial, distinction between what he can and what he cannot prove.

[14] Special thanks here to Tim Williamson for discussion of the preceding few paragraphs.

[15] Dummett (1978c) makes a similar point. Thus, the ineradicability of vagueness is closely related to so-called 'higher-order' vagueness, the vagueness not only of such predicates as 'ξ is red' but such predicates as 'ξ is definitely red', 'ξ is definitely definitely red', and so on. Elsewhere, I argue that a proper understanding of higher-order vagueness rests upon much the same insights about the logic of "Definitely" which I have been discussing here (Heck, 1993).

[16] Special thanks here to Tim Williamson for discussion of the preceding few paragraphs.

5 Whence the Contradiction

The most natural suggestion to make at this point is that Evans is appealing, not to the axiom (Det), but to the following rule of inference:

(Det*) $A \vdash \Delta A$

There is a corresponding rule for "Definitely":

(V) $A \vdash \Box A$

It is important to distinguish these rules from analogues of the familiar modal rule of Necessitation, a version of which (NecΔ) was mentioned above. The rule of 'Definitization', for example, states that, if A is valid, so is '$\Box A$'. It is what is sometimes called a *rule of proof*. But the rule (V) is not intended as a rule of proof: It states instead that '$\Box A$' may be inferred from A; or, again, that if A is derivable *from certain premises*, so is '$\Box A$'. Such a rule is surely valid: For, if A is true, then "Definitely: A" is indeed true; or, as we may also put it, if A is true, then A is *definitely* true. That is all that is required of a valid rule of inference, that its conclusion be true whenever its premises are true.[17] And since '$\Box A \to \Delta A$' is valid, the validity of (Det*) follows from that of (V).

The point of introducing the rule of inference (Det*) is to get the effect of (Det) without its disadvantages. Hence, we must renounce the application of (Det*) within conditional proof: For, using conditional proof, (Det) can easily be derived with the help of (Det*). Similarly, we must renounce the application of (Det*) in proofs by cases and *reductio ad absurdum*: For, using either of these forms of argument, we should again be able to demonstrate the validity of 'ΔA' without appeal to the transparency of 'Δ'. Just as in the case of (Det), we should be without an argument that there are no vague objects. More generally, we must renounce appeal to (Det*) within subordinate deductions, within, that is, deductions from premises which may subsequently be discharged. I shall refer to rules such as (Det*) as *rules of deduction*, in order to distinguish them from other sorts of rules of inference, which are applicable in both main and subordinate deductions. I hereby interpret Evans as having assumed the validity of the rule (Det*) as a rule of deduction.

With the rule (Det*) in hand, we can complete Evans's formal proof:

[17] Those attracted to fuzzy logics often consider rules which require more, namely, that the value of the conclusion should be at least as high as that of the premise. There is no reason not to consider such rules, if one wishes, but, by the same token, there is no reason not to consider rules of the sort I'm considering here.

1. $\nabla(a = b)$ Premise

2. $\neg\nabla(a = a)$ (R), (C)

3. $a = b \wedge \nabla(a = b) \to \nabla(a = a)$ (LL)

4. $a \neq b$ (1,2,3) TF

5. $\Delta(a \neq b)$ (4) Det*

6. $\neg\nabla(a = b)$ (5) D

7. $\nabla(a = b) \wedge \neg\nabla(a = b)$ (1,6) TF

Contradiction. Because we have been forced to renounce application of (Det*) within proofs by *reductio*, however, we cannot infer '$\neg\nabla(a = b)$'. Even granted that '∇' is transparent, we cannot prove, *via* (Det*), that '$\Delta(a = b)$' is valid.[18]

Assuming that the (amended) formal argument Evans has presented is one his opponent must accept, the only remaining problem with his full argument is now the final step: That by which he passes from the conclusion that '$\nabla(a = b)$' is unsatisfiable, to the ultimate conclusion that there can be no vague objects. I shall disucss this step after I discuss Evans's appeal to Leibniz's Law.

6 Vague Objects and Leibniz's Law

Noting the utility the notion of a rule of deduction has, one might well seek to defend the satisfiability of '$\nabla(a = b)$' by denying Evans's appeal to the Indiscernibility Principle in the form in which it is required for

[18]We can give models for this theory as follows. Let the underlying structure of the models be that for a quantified version of S4 in which '$a = b \to \Box a = b$' is valid. (The details do not matter for present purposes.) Instead of taking truth in a model to be truth at some 'actual' world, we take truth to be truth in *all* worlds—the semantics is thus supervaluational. Define 'ΔA', as usual, as '$\Box A \vee \neg\Box A$'. (Eq) is obvious; take (D) as the definition of the dual. One may verify that (LL) holds.

Suppose A is true. Then A is true at all worlds; so '$\Box A$' is true at all worlds; so 'ΔA' is true at all worlds; so 'ΔA' is true. Hence, (V) is valid.—Note, however, that '$A \to \Delta A$' is not valid, since, though it cannot be false at *every* world, it may be false at some.

Evans's proof shows that '$\nabla(a = b)$' is unsatisfiable. However, '$\Delta a = b$' is not valid. Let there be two worlds, w and w'. Take w' accessible to w, though not conversely. Let '$a = b$' be false at w; true, at w'. Then '$\Delta a = b$' is not true at w and so is not (absolutely) true. What we can do is derive a contradiction from '$\nabla(a = b)$'. It follows, of course, that no sentence of the form '$\neg\Delta a = b$' can ever be true, i.e., that '$\nabla(a = b)$' is unsatisfiable.

his argument. In this section, I shall look at the prospects of such a move, concluding that they are poor.

Earlier, I recorded this principle in the form:

(LL) $x = y \wedge \Phi(x) \rightarrow \Phi(y)$

Given the transparency of '$\Box$' and 'Δ', it is then easy to derive the two schemata:

$$a = b \rightarrow \Box(a = b)$$
$$a = b \rightarrow \Delta(a = b)$$

Both of these principles have been questioned in the literature (Garrett, 1988). Consider, for instance, the latter: Suppose that '$a = b$' is neither true nor false; then '$\Delta(a = b)$' is at least not true; hence, '$a = b \rightarrow \Delta(a = b)$' is plausibly not true. No instance of this conditional can possibly be false, since, if '$a = b$' is true, so is '$\Delta(a = b)$': But it does not follow that the conditional is valid. And, if that is right, (LL) itself is not a valid schema.

The Indefinitist is still going to need *something* to take the place of (LL), for something must play the logical role played by (LL); so she may adopt, instead of (LL), the rule of deduction (LL*):

$$a = b, Fa \vdash Fb$$

After all, if a is b, then a and b must share all their properties: So, if it is true that $a = b$ and it is true that, say, ΔFa, it must also be true that ΔFb.

If appeal to this rule *alone* is allowed, Evans's proof fails. For we cannot derive, from the assumptions that Fa and $\neg Fb$, that $a \neq b$. That is, we cannot prove, as a derived rule, (II*):

$$Fa, \neg Fb \vdash \neg a = b$$

We might try to do so as follows:

[1]	(1)	Fa	Premise
[2]	(2)	$\neg Fb$	Premise
[3]	(3)	$a = b$	Premise
[1,3]	(4)	Fb	(1,3)LL*
[1,2,3]	(5)	$Fb \wedge \neg Fb$	(2,4)TF
[1,2]	(6)	$a \neq b$	(3,5)RAA

But this proof by *reductio* is invalid, since appeal to (LL*), a rule of deduction, is invalid within subordinate deductions. In principle, then, one may accept the validity of (LL*) while denying that of (II*). And if only the rule of deduction (LL*) is accepted as valid, we cannot show that '$\nabla(a = b)$' is unsatisfiable. Denial of the validity of (LL) and its replacement by (LL*) will block the derivation of a contradiction from '$\nabla(a = b)$'.[19]

However, the derivation of the contradiction does not require appeal to (LL): It requires only appeal to the rule (II*). One needs to be able to derive '$a \neq b$' from '$\neg\nabla(a = a)$' and '$\nabla(a = b)$', and the rule (II*) licenses this inference. To defend the view that sentences of the form '$\nabla(a = b)$' are satisfiable, one must therefore deny not only that (LL) is valid but also that (II*) is valid. But it is difficult to see on what ground (II*) is to be rejected. It is one thing to argue against (LL), as above, that if 'Fa' is true and 'Fb' is neither true nor false, then '$a = b$' need not be false, but need only fail to be true. It is another thing entirely to suggest that, if it is indeterminate whether a is b, there might, in fact, be some predicate '$F\xi$' such that 'Fa' is true although 'Fb' is *false*.—Intuitively, it is one thing to suggest that, if it is indeterminate whether a is b, then it might be *indeterminate* whether they 'share all their properties'; it is another to suggest that they might *not* share them.

The crucial disanalogy between the rejection of (LL) and the rejection of (II*) is that the argument against (II*) depends upon our adopting a very specific interpretation of the operators as '$\Box$' and 'Δ'.[20] One may wish to reject (LL), for reasons like those mentioned above: That ground for the rejection of (LL) does not depend upon the presence of

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[19]We give models for such a theory. Let the underlying structure be that for a quantified version of S5, *without* the assumption that '$a = b \rightarrow \Box a = b$' is valid. (Again, the details do not at present matter.) We require only that, if '$F\xi$' does not contain '$\Box$', then, if '$a = b$' is true at a world, '$Fa \equiv Fb$' is also true at that world. We again define truth as truth in all worlds: Hence, (V) is valid.

It is straightforward to prove, by induction on the number of occurences of '$\Box$' in '$F\xi$', that '$a = b \rightarrow (Fa \equiv Fb)$' is valid, for *any* predicate '$F\xi$'. Hence, if '$a = b$' and 'Fa' are (absolutely) true, then, since '$a = b \wedge Fa$' is true at all worlds, so must 'Fb' be true at all worlds. Hence, (LL*) is valid.

We show simultaneously that (II*) fails—and so is independent of (LL*)—and that '$\neg a = b \wedge \neg\Box\neg a = b$' is satisfiable. Let there be two worlds w and w'. Let '$a = b$' be true at w; false, at w'. Then, of course, '$\Box(a = a)$' is true at both w and w'; and '$\neg\Box(a = b)$' is also true at both w and w'. Both '$\Box(a = a)$' and '$\neg\Box(a = b)$' are then (absolutely) true. So '$\neg a = b \wedge \neg\Box\neg a = b$' is absolutely true and so is satisfiable. Moreover, since '$\neg a = b$' is not absolutely true (it is false at w), (II*) fails.

[20]This sentence was a bit mangled in the original version (RGH, 2011).

such operators as '$\Box$' and 'Δ'; it requires only the meta-linguistic claims that '$a = b$' itself may be neither true nor false and that, if it is, '$\Box a = b$' is not true. The rejection of (II*), on the other hand, depends upon the presence of such operators as '$\Box$' and 'Δ' within the language, upon the assumption that they are transparent, and upon assumptions about the truth-values of sentences containing them. If we assume the transparency of 'Δ', so that '$\Delta F\xi$' is a legitimate substituend in (II*) if '$F\xi$' is, if we so explain 'Δ' that, if 'Fa' is neither true nor false, then 'ΔFa' is false, *and* if we assume that '$\nabla(a = b)$' is satisfiable, then we shall find ourselves compelled to reject (II*). Evans shows us why. But it is not clear that we are entitled simultaneously to make all three of these assumptions.

Again, the validity of (II*) is inconsistent only with the following trio of claims: First, that '$a = b$' might be neither true nor false; Second, that '$\Box A$' is false, if A is not true (or that 'ΔA' is false, if A is neither true nor false); and, Third, that '$\Box$' and 'Δ' are transparent operators. But it is unclear that one is entitled simultaneously to make stipulations about the transparency of an operator *and* about the truth-values of sentences which contain it. On the contrary, one ought to explain the operator (settle the truth-conditions of sentences containing it) and then *ask* whether, so explained, it is transparent. Or, conversely, one ought decide upon the transparency of the operator and then *ask* how, consistently with its transparency, its truth-conditions may be fixed. To answer either of these questions, one must make reference to that form of Leibniz's Law which is properly taken to be valid *prior* to the introduction of the new operator, for it is only by reference to the laws governing identity that we have any purchase on the notion of transparency in the first place: Hence, *if* the new operator is transparent, it will be subject to whatever form of Leibniz's Law is valid for sentences which do not contain it. Thus, since (II*) appears to be valid, so long as permissible substituends contain neither '$\Box$' nor 'Δ', it must remain valid once we admit predicates containing these operators, if we wish these operators to be transparent.[21]

The foregoing does, I think, constitute a sound argument for the validity of (II*). But, even if the argument is not entirely successful, it may yet serve its purpose, since the question is not so much whether (II*) is

[21]Perhaps the following, intuitive way of making this point will be helpful to some: If '$\Box F\xi$' is not subject to the laws of identity which govern '$F\xi$' itself, then '$\Box F\xi$' is not true or false *of objects* in the same sense in whcih '$F\xi$' is.

valid as whether the assumption that it is begs the question against one who wants to maintain that '$\nabla(a = b)$' is satisfiable. And our discussion clearly shows, it seems to me, that it does not. It is, of course, the whole point that if (II*) is valid, and if predicates containing '$\Box$' or 'Δ' are transparent, then '$\nabla(a = b)$' is not satisfiable. But that does not imply that the assumption of (II*) begs the question: If it did, it would be impossible to argue at all.

7 Indefinitism (Partially) Explained, and Defended against Evans

I resume my assumption that Leibniz's Law is valid in its classical form

$$x = y \wedge \Phi(x) \to \Phi(y)$$

even if vague objects are in the domain. One who wishes to defend the existence of vague objects must therefore accept Evans's derivation of a contradiction from '$\nabla(a = b)$', must grant that there neither are nor could be any true sentences of the form '$\nabla(a = b)$', i.e., that '$\nabla(a = b)$' is unsatisfiable. Nothing, however, in Evans's argument requires her to grant that '$\Delta(a = b)$' is valid.

Whether Evans's argument shows that there can be no vague objects may now seem to be but a terminological question. We know what Evans's argument shows and what it does not: It does show that '$\nabla(a = b)$' is unsatisfiable; it does not show that '$\Delta(a = b)$' is valid. If we identify the view that there are vague objects with the view that there are (or might be) true sentences of the form '$\nabla(a = b)$', then Evans has shown that there are no vague objects. If, on the other hand, we identify the view that there are vague objects with the view that '$\Delta(a = b)$' is not valid, then he has not. But the dispute ought not be allowed to become merely verbal.

Evans certainly took himself to be arguing an ontological point, namely, that there can be no vague objects. Any argument for such a conclusion must rest upon some characterization of the nature of the dispute. Evans's view can only have been that one who maintains that there are vague objects must hold that '$\nabla(a = b)$' is satisfiable. Indeed, he opens his paper by saying that his opponent takes it to be "a *fact*" that some identity-statements are of indeterminate truth-value. This is a natural way to understand the view. Nonetheless, in the remainder of this section, I shall be arguing that Evans is mistaken that the Indefinitist is committed to the claim that '$\nabla(a = b)$' is satisfiable, for the following reason: There is a view, which is plausibly committed to the existence of vague objects, which is *not* committed to the satisfiability of '$\nabla(a = b)$'. I shall thus argue that Evans's claim to have proven that there are no vague objects fails, even though the (formal) argument he gives is valid, since we need not accept his characterization of his opponents' view.

The view that there are no vague objects presents us with a picture: The 'boundaries' of any given object are perfectly determinate; because its boundaries are determinate, the identity of the object (i.e., its identity with and distinctness from other objects) is also perfectly determinate. Every identity-statement not only is, but must be, either (definitely) true or (definitely) false. It is therefore natural to characterize Definitism as the view that '$\Delta(a = b)$' must be true, that is, that it is *valid*. So, if Indefinitism is the denial of Definitism, it is the denial of the claim that every identity-statement must be of determinate truth-value. If this is right, Indefinitism is committed to no more than the invalidity of '$\Delta(a = b)$', and Evans's argument fails to show that it is logically incoherent.

Indefinitism was implicitly supposed by Evans to be committed to the claim that it might be neither true nor false that certain objects are identical, that this might itself be a "fact". It is natural to suppose that, on this view, identity-statements are supposed to have some *truth-value* other than True or False, to read '$\nabla(a = b)$' as "It is neither true or false that $a = b$", and so to take '$\nabla(a = b)$' to be *true* if '$a = b$' has one of the intermediate truth-values. This sort of view is committed to the satisfiability of statements of the form '$\nabla(a = b)$', and I have argued above that Evans's argument refutes all such views. Fortunately for those with Indefinitist leanings, however, there is another alternative to Definitism: To reject the characterization of "Definitely" as a many-valued truth-functional operator and reject the underlying assumption that languages containing vague terms or vague predicates admit of a many-valued semantics of any kind.[22] Such an alternative rejects the claim that every statement objectively has some one of *however many* truth-values, thereby departing radically from the view that every identity-statement is either true or false. Indefinitism is not the view that there is something other than True or False for

[22]The main goal of Pelletier's paper is to establish this conclusion by generalizing Evans's argument (Pelletier, 1989). It is difficult for me to understand, though, why he takes Indefinitism to be committed to a many-valued semantics.

identity-statements to be: It is the view that our model of truth and falsity, as objective properties of sentences, whose possession of one of the truth-values is independent of our cognitive capacities, does not apply to identity-statements containing names of vague objects. The view that there are not two, but three (or more), truth-values can only seem comparatively familiar.

Such views are not entirely unknown, though, and we may take, as a model for the sort of semantic theory an Indefinitist must provide, that employed in contemporary developments of the Intuitionistic philosophy of mathematics.[23] Intuitionists reject the principle of bivalence and so deny that every statement is determinately either true or false. Yet, according to Intuitionism, no statement can be *neither* true nor false: The Intuitionist's view is emphatically *not* that, instead of two truth-values, there are many. The view is not that some statements are true, others false, and yet others neither true nor false, but rather that mathematics does not merit the kind of objectivity we are inclined to accord to it: In mathematics, on this view, we may speak of what is true only in terms of what is provable; of what is provable, only in terms of what we can, in principle, prove. Intuitionism rejects any notion of truth according to which the truth or falsity of a mathematical statement is independent of our epistemic capacities; hence, the notion of a statement which is *objectively* 'neither true nor false'—one which could neither be proven nor refuted by any intiutive proof—is, at best, not one for which Intuitionists have any use and would be regarded, by many

[23]There is a formal similarity between Indefinitism and Intuitionism which is worth mentioning: We *can* prove '$\neg\Box\nabla(a = b)$', i.e., "It is not definitely true that it is indeterminate whether a is b". Evans's proof shows that '$\nabla(a = b) \rightarrow a \neq b$' (for we have yet to apply (V) and so may take Evans's proof as a conditional proof). We then argue as follows:

1.	$\nabla(a = b) \rightarrow a \neq b$	Evans's proof
2.	$\Box[\nabla(a = b) \rightarrow a \neq b]$	Definitization
3.	$\Box\nabla(a = b) \rightarrow \Box a \neq b$	Distribution CHECK
4.	$\Box\nabla(a = b) \rightarrow \Delta(a \neq b)$	(3),$\Delta/\Box$ equivalences
5.	$\Delta(a \neq b) \rightarrow \neg\nabla(a = b)$	C
6.	$\Box\nabla(a = b) \rightarrow \neg\nabla(a = b)$	(4,5) TF
7.	$\Box\nabla(a = b) \rightarrow \nabla(a = b)$	T
8.	$\neg\Box\nabla(a = b)$	(6,7) TF

The Indefinist's acceptance of '$\neg\Box\nabla(a = b)$' may well be compared to the Intuitionist's acceptance of the validity of '$\neg\neg(A \vee \neg A)$'.

Intuitionists, as dubiously intelligible.[24]

Whether Indefinitism is so much as coherent thus reduces to the question whether the kind of semantic theory, the sort of concept of truth, which would accord with such a view is itself coherent. The analogy with Intuitionism is meant to suggest no more than that the formulation of an alternative to Definitism—an alternative to the view that the vagueness of identity-statements is always due to our epistemic and linguistic limitations—requires the development of a view for which Intuitionism is our best model. But, for all that it merely points in the direction of an understanding of Indefinitism, the analogy does at once explain why the position seems so threatening: Indefinitism asks us to accept, not just the intelligibility, but the actual applicability to our thought, of an epistemically constrained notion of truth. Indefinitism is a species of *anti*-realism.

It is for this reason that I have spoken, almost exclusively, of the view that there are vague objects, and only rarely of the view that vagueness is 'in reality', and not at all of the 'reality of vagueness'. As we have seen, Evans's argument fails to establish that there are no vague objects, since it establishes only that '$\nabla(a = b)$' is unsatisfiable, and Indefinitism is consistent with this claim. However, Evans's argument probably does establish, or so I am now suggesting, that the view that there are vague objects is incompatible with a Realist treatment of statements containing names of them. Evans himself might have concluded from this that there are no vague objects, but it is not for *logic* to decide such matters.[25]

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[24] I am here drawing, of course, on the discussion in Michael Dummett's "The Philosophical Basis of Intuitionistic Logic" (Dummett, 1978a).

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